量子上同调学术研讨会
-- 第一期会议

时间：**2023 年 11 月 5 日**

地点：南方科技大学 理学院 **M1001** 报告厅

　　　#腾讯会议：744-290-404

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　　　胡晓文 大湾区大学

　　　何伟强 中山大学

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会议日程安排：

<table>
<tr><td colspan="2" rowspan="1">时 间</td><td>报告题目</td><td>报告人</td></tr>
<tr><td rowspan="3">上午</td><td>8：30-10：00</td><td>Fundamentals and an overview</td><td>胡晓文(大湾区大学)</td></tr>
<tr><td>10：00-10：30</td><td colspan="2">茶歇</td></tr>
<tr><td>10：30-12：00</td><td>Equivariant cohomology and the localization theorem</td><td>游磊（香港科技大学）</td></tr>
<tr><td colspan="2">12：00-14：00</td><td colspan="2">午餐</td></tr>
<tr><td rowspan="2">下午</td><td>14:00-15:30</td><td>Formal aspects of quantum cohomology</td><td>卢经纬（中山大学）</td></tr>
<tr><td>15:30-16:00</td><td colspan="2">茶歇</td></tr>
<tr><td></td><td>18　00</td><td></td><td></td></tr>
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Talk 1 by 胡晓文 (1.5 hours): *Fundamentals and an overview.*

- Goal: First give a quick introduction to the construction of moduli space of stable maps, and virtual fundamental cycles. Then give an overview of [Iri23]. Discuss some related problems in algebraic geometry.

Talk 2 by 游磊 (1.5 hours): *Equivariant cohomology and the localization theorem.*

- Goal: Let $T = (\mathbb{C}^\times)^n$ be a complex torus. For a T-space X, introduce the T-equivariant cohomology (with $\mathbb{Q}$-coefficients) of X in terms of the Borel construction. Show that the equivariant cohomology has the similar functoriality as the ordinary cohomology, especially the pushforward ([AtB84, §2], [Ric22, §8.2]). Then show the localization theorem [AtB84, Thm. 3.5]

$$H_T^*(X) \otimes_{H_T^*(pt)} \mathrm{Frac}\big(H_T^*(pt)\big) \cong H^*(X^T) \otimes_{\mathbb{Q}} \mathrm{Frac}\big(H_T^*(pt)\big),$$

  and then deduce the Atiyah-Bott integration formula [AtB84, (3.8)]

  (2)
  $$\int_X \psi = \sum_\alpha \int_{F^\alpha} \frac{\iota_\alpha \psi}{e_T(N_\alpha)} \in \mathrm{Frac}\big(H_T^*(pt)\big),$$

  where F_α runs over the connected components of the fixed loci X^T, and $\psi \in H_T^*(X)$. Finally, give applications of Atiyah-Bott integration formula (2). The speaker can find examples from [CK99, §9.1.2, §9.1.3] and [Ric22, Chap. 9].

- Ref: See [Hsi75, P. 39, Prop. 2] for the localization theorem in the circle group case, and the general case follows by taking a dense circle in T. For the approach via the Cartan model of equivariant cohomology, see [Tu20, Chap. V].

Talk 3 by 卢经纬 (1.5 hours): *Formal aspects of quantum cohomology.*

- Goal: Cover Sections 2.1-2.4 of [Iri23]. Especially, explain the construction of the quantum D-module. Prove [Iri23, (2.6)-(2.8)]. Define equivariant quantum cohomology and the equivariant quantum D-module.

REFERENCES

[AtB84] Atiyah, M. F.; Bott, R. The moment map and equivariant cohomology. Topology 23 (1984), no. 1, 1–28. 1, 2

[CK99] Cox, David A.; Katz, Sheldon. Mirror symmetry and algebraic geometry. Mathematical Surveys and Monographs, 68. American Mathematical Society, Providence, RI, 1999. 1

[Hsi75] Hsiang, Wu-yi. Cohomology theory of topological transformation groups. Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 85. Springer-Verlag, New York-Heidelberg, 1975. 2

[Iri23] Iritani, Hiroshi. Quantum cohomology of blowups. arXiv:2307.13555 (2023). 1, 2, 3

[Ric22] Ricolfi, Andrea T. An Invitation to Modern Enumerative Geometry. Vol. 3. Springer Nature, 2022. 1

[Tu20] Tu, Loring W. Introductory lectures on equivariant cohomology. With appendices by Tu and Alberto Arabia. Annals of Mathematics Studies, 204. Princeton University Press, Princeton, NJ, 2020. 2