Abatracts

Parabolic implosion in dimension 2

Matthieu Astorg (Université d'Orléans)

Parabolic implosion is the study of the bifurcation phenomena related to perturbations of a holomorphic map which has a fixed point with multiplier 1 (or more generally a root of unity). We will review the now classical theory in dimension one, and present a recent generalization of parabolic implosion for holomorphic maps in dimension 2, with a fixed point whose differential is the identity.

Entropy for hyperbolic holomorphic foliations

François Bacher (Université de Bourgogne)

Dinh, Nguyên and Sibony have developed an ergodic theory for foliations by hyperbolic Riemann surfaces. For that purpose, they considered some kind of canonical dynamics, the time of which is measured by the leafwise Poincaré metric. This way, one can define an entropy "à la Bowen" for such foliations. One can also do the same for the entropy of a harmonic measure. During this talk, I will discuss these various notions of entropy and some recent results obtained for singular foliations and for suspensions.

Bifurcation loci as shadows of Julia sets

Fabrizio Bianchi (Università di Pisa)

We show that the bifurcation loci of holomorphic families of rational maps (and more generally of endomorphisms of P^k in any dimension) can be seen as the projection of a suitably defined Julia set for a self-map of a higher-dimensional space. A similar statement holds for the bifurcation currents, and permits the study of the bifurcation loci and currents using dynamical tools typical of phase spaces. This is a joint work with F. Berteloot and T.-C. Dinh.

Rigidity near invariant Diophantine tori

Abed Bounemoura (Universit´e Paris Dauphine)

Abstract: We will discuss notions of rigidity, both formal and topological, near an invariant Diophantine torus for a smooth or analytic Hamiltonian system. In particular, for an analytic system, we will show that if the dynamics is topologically linearizable, then it is analytically linearizable.

Almost reducibility of quasiperiodic cocycles in ultradifferentiable classes

Maxime Chatal (Institut de Mathématiques de Jussieu - Paris Rive Gauche (IMJ-PRG))

To study the dynamics of quasiperiodic cocycles, one can investigate their reducibility or almost reducibility. Reducibility means that the system is conjugate to a constant cocycle, while almost reducibility means that it can be conjugated arbitrarily close to a constant cocycle. The analytic case is well understood under Diophantine conditions on the frequency vector. Here, we consider the more general setting of ultradifferentiable classes and frequency vectors satisfying an adapted Brjuno-type condition.

Real almost reducibility of quasi-periodic cocycles

Claire Chavaudret (Institut de Mathématiques de Jussieu - Paris Rive Gauche (IMJ-PRG))

Quasi-periodic cocycles are transfer matrices of linear systems with quasi-periodic coefficients. As in Floquet theory, one tries to reduce such a system to an autonomous one, which is not always possible in the quasi-periodic case. Eliasson has shown that, in a perturbative analytic framework (if the frequency vector is Diophantine, and if the coefficient matrix is analytically close to a constant), linear quasi-periodic systems are almost reducible, that is, they can be conjugated to a system arbitrarily close to a constant. This was extended to other regularity classes.

In this talk, I will present a joint work with M.Chatal and H.Eliasson, where we show that a real valued quasi-periodic cocycle, in the C^∞ class, if almost reducible (through complex conjugations), can also be « almost reduced » to a sequence of real cocycles with real conjugations, with just one period doubling. In other words, there is some algebraic rigidity about almost reducibility.

Higher Bessel Product Formulas: Explicit Examples of Multiplication Kernels

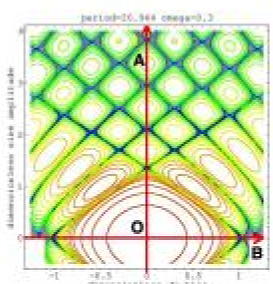
Ilia Gaiur (Institut des Hautes Études Scientifiques)

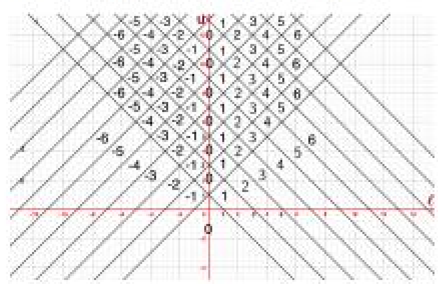
Higher Bessel functions are the solutions to the quantum differential equations for $\mathbb{P}^{N-1}$. These functions are connected to the periods of the Dwork families via the Laplace transform, and the functions themselves are exponential integrals. In my talk, I will show how product formulas for these irregular special functions lead to other geometric differential equations associated with higher-dimensional families of algebraic varieties. I will discuss the geometric and algebraic properties of the periods for these families and later provide further perspectives on these correspondences.

On dynamical systems on torus modeling Josephson junction and Heun equations

Alexey Glutsyuk[1]

In 1962 B.Josephson (Nobel Prize 1973) predicted a tunnelling effect related to a system of two superconductors separated by a narrow dielectric: the so-called Josephson junction. This effect is the existence of a supercurrent crossing the junction and governed by equations discovered by Josephson. The overdamped Josephson junction is modeled by a family of differential equations on two-dimensional torus that depends on three parameters: the abscissa B; the ordinate A and a fixed frequency ω of exterior forcing. The corresponding rotation number is a function of (B,A). The phase-lock areas are those its level sets that have non-empty interiors. They exist only for integer rotation numbers, as was discovered by V.M.Buchstaber, O.V.Karpov and S.I.Tertychnyi. Buchstaber and Tertychnyi have shown that the model is equivalent to a family of second order linear differential equations: special double confluent Heun equations. Each phase-lock area L_r with rotation number $r \in \mathbb{Z}$ is a garland of infinitely many domains separated by points: white domains in the left figure. It was shown by Yu.Bibilo and the speaker that those separation points that do not lie on the abscissa axis lie on the vertical line with abscissa $r\omega$. This was done by using relations to linear equations and isomonodromy deformations.





We present a survey of results and open questions. We show that as $\omega \to 0$, the phase-lock area portrait in the rescaled parameter plane $\mathbb{R}^2_{\ell,u}$, $\ell := \frac{B}{\omega}$, $u := \frac{A-1}{\omega}$, i.e., in a $O(\omega)$-neighborhood of the point $(0,1) \in \mathbb{R}^2_{B,A}$, converges to a parquet. Namely, each phase-lock area L_r converges to a union of squares with diagonals of length 2 lying in the vertical line with abscissa r and an infinite domain: a half-strip for $r \neq 0$, respectively a sector for $r = 0$. See the right figure, where the domains forming the limit of each phase-lock area L_r are marked by r.

[1]Research is supported by the MSHE RF GZ project and by grant No. 24-7-1-15-1 of the Theoretical Physics and Mathematics Advancement Foundation BASIS.

Persistence of complex limit cycles and simultaneous uniformization: results and problems

Yulij Ilyashenko (Moscow State University / Higher School of Economics)

Petrovsky-Landis strategy in attempt to solve the Hilbert 16th problem. Persistence problem for limit cycles. Heteroclinic points and the Kupka-Smale property of the Henon maps: a successful application of the Petrovsky-Landis strategy. Simultaneous uniformization of the families of analytic curves: positive and negative results. Simultaneous uniformization implies persistence of limit cycles. Necessary and sufficient condition for simultaneous uniformization by Kleinian groups.

Spectral properties of Schrödinger operators with quasi-periodic potential and Aubry-Mather theory

Konstantin Khanin — (Beijing Institute of Mathematical Sciences and Applications (BIMSA))

Schrödinger operators with quasi-periodic potentials were intensively studied in the last few decades. Their spectral properties depend on the value of the coefficient in front of the potential, so called. coupling constant. For small values of the coupling constant the spectrum is absolutely continuous, while for large coupling constants the spectrum is pure point. Natural families of Schrödinger operators with quasi-periodic potentials appear in the context of the Aubry-Mather theory. In this setting there are no coupling constants. Instead operators depend on the nonlinearity parameter for related area-preserving maps. We shall discuss the transition from the absolutely continuous to the pure point spectrum for such families of Schrödinger operators.

Divergence of formal maps in CR geometry

Bernhard Lamel (University of Vienna)

While CR maps on minimal manifolds have nice convergence properties, if the source is nonminimal, divergent maps will occur. We review some of the important results, and talk in detail about about our joint work with Kossovskiy and Stolovitch on realizing formal power series maps as Taylor expansions of smooth CR maps, utilizing multisummability methods.

Kolmogorov widths asymptotics in different holomorphic function spaces

Stephanie Nivoche (Université Côte d’Azur)

In order to study the complexity of function spaces, we estimate the asymptotics of the sequence of Kolmogorov m-widths of compact sets of holomorphic functions in Bergman spaces, Hardy spaces and Fock spaces. The technics use some concentration results for the eigenvalues of a certain family of Toeplitz operators and an exhaustion procedure by special holomorphic polyhedra, originating from Bishop's work.

Singularities of the Painlevé monodromy manifolds

Victor Novokshenov
Institute of Mathematics, Ufa Federal Research Center RAS, Russia

The nonlinear Painlevé differential equation for the function $u(z)$ is integrated using the isomonodromic deformation method. It consists in choosing an auxiliary linear system of two ODEs of first order in the variable $\lambda \in \mathbb{C}$ such that its deformation with respect to z and $u(z)$ as parameters preserves monodromy when traversing singular points. Sometimes there are no finite singular points in such a system, then the Stokes matrices that arise when walking around infinity along λ are chosen as monodromy data. Thus, the monodromy data play the role of integrals of motion or conservation laws for the corresponding Painlevé equation.

The set of monodromy data forms a *monodromy manifold* of complex dimension 2, since all second-order Painlevé equations and their solutions $u(z)$ depend on two arbitrary constants. There is a one-to-one correspondence between the points of the monodromy manifold and solutions of the Painlevé equation. So, by describing the structure of the monodromy variety, we can describe the structure of the space of functions $u(z)$.

Painlevé equations of the first and second types are considered

$$\begin{array}{ll} P_1 & u'' = 6u^2 - z, \\ P_2 & u'' = zu - u^3. \end{array}$$

Their monodromy manifolds are given by

$$\mathcal{P}_1 = \{(s_1, s_2, s_3) \in \mathbb{C}^3 \,|\, F = s_1 s_2 s_3 - s_1 - s_2 + 1,\ F = 0\}. \tag{1}$$

$$\mathcal{P}_2 = ((s_1, s_2, s_3) \in \mathbb{C}^3 \,|\, F = s_1 s_2 s_3 - s_1 - s_2 - s_3,\ F = 0). \tag{2}$$

For other Painlevé equations, the monodromy manifold is also realized as an affine cubic Klein surface in $\mathbb{C}^3$. They also arise in other areas of mathematics and physics. In particular, Sklyanin algebras arising from the solution of the Yang-Baxter equations are parameterized by the variety $\mathcal{P}_2$ [2].

We relate the geometry of monodromy manifolds to the distribution of singularities of solutions to the Painlevé equations. It is well known that all solutions of the equations P_1 and P_2 are meromorphic functions in the complex plane z. The distribution of their poles was studied in the early works of P. Boutroux [1], where classes of so-called *truncated solutions* (tronqueé solutions) were found. They correspond to the absence of poles at infinity in critical sectors bounded by rays $\arg z = \pi k/6$, $k = 1, \ldots, 6$.

A solution is called k-truncated if it has no poles on k critical rays.

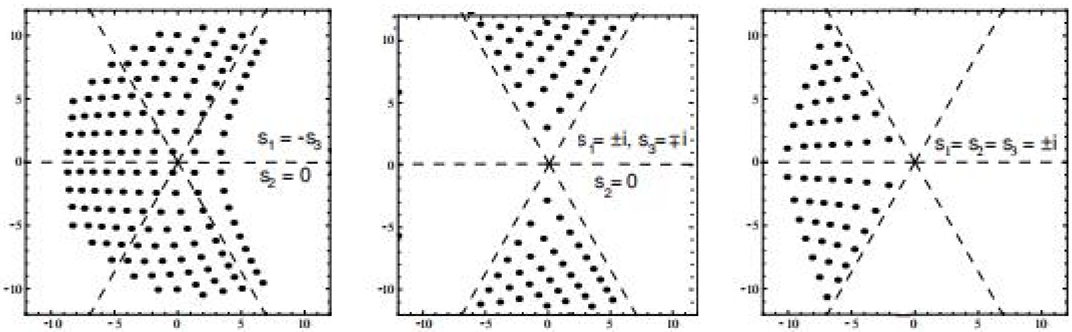


Figure 1: Poles of truncated solutions to P_2, 1-truncated (left), 2-truncated (center) and 3-truncated (right) and their monodromy data. The borders of critical sectors are shown in dashed lines.

The singularities of manifolds (1) and (2) are determined by the equation $dF = 0$. It turns out that these submanifolds of singularities parametrize truncated solutions of the Painlevé equations [3].

Theorem. *All 1-truncated solutions of the equations P_1 and P_2 correspond to one-dimensional singularity submanifolds $\mathcal{P}_1$ and $\mathcal{P}_2$. 2- and 3-truncated solutions correspond to zero-dimensional submanifolds of singularities.*

References

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[2] M. Mazzocco, L. Chekhov and V. Rubtsov, Painlevé Monodromy Manifolds, Decorated Character Varieties, and Cluster Algebras, Inter. Math. Research Notices, 2016 (24), (2017), 7639 – 7691.

[3] V. Yu. Novokshenov, Special sotutions of the first and second Painlevé equations and singularities of the monodromy data, Proc. Steklov Inst. Math. (Suppl.), 281, suppl. 1, (2013), 105–117.

Inverse monodromy problems on elliptic curve

Vladimir Poberezhny (Higher School of Economics)

Many foundational problems and results concerning classical inverse monodromy problems on the Riemann sphere admit natural generalizations to the setting of Riemann surfaces. In this broader context, the Riemann–Hilbert problem can be reformulated as the problem of constructing logarithmic connections on semistable vector bundles of degree zero with prescribed monodromy and singularities. The classical negative result in general, as well as the positive solution for irreducible monodromy representations, largely extend from the case of the Riemann sphere. At the same time, isomonodromic deformations of such connections are deeply related to the theory of integrable systems and have become a central object of study in mathematical physics. However, many results known for the classical Riemann–Hilbert problem have not yet been successfully generalized to arbitrary Riemann surfaces. For instance, explicit solutions are still unknown even for commutative monodromy representations, and a rigorous proof of the Painlevé property for isomonodromic deformations on Riemann surfaces remains an open problem. Elliptic curves occupy an intermediate position between the Riemann sphere and general Riemann surfaces. Although they are curves of positive genus, they nevertheless admit an explicit classification of vector bundles and a concrete description of sections in terms of classical theta functions. In this talk, we present an overview of known results and current challenges concerning inverse monodromy problems on elliptic curves. In particular, we describe an explicit solution of the Riemann–Hilbert problem on an elliptic curve for two-dimensional commutative monodromy representations.

Measures of large entropy in holomorphic dynamics

Karim Rakhimov (V.I. Romanovskiy Institute of Mathematics)

Let f be a holomorphic endomorphism of $\mathbb{P}^k(\mathbb{C})$ of algebraic degree d. The Julia set J of f is the support of the unique invariant measure μ of maximal entropy $k\log d$. De Thélin and Dinh proved that every invariant measure whose entropy is larger than $(k-1)\log d$ is also supported on J. Their proof relies crucially on the existence of the Green current and uses a delicate induction based on the successive self-intersections T^j of the Green current T. Consequently, it is unclear how to extend this result to more general non-algebraic settings where a dynamical Green current may not exist.

In this talk, I will present an extension of this result to the more general setting of polynomial-like maps of large topological degree. The proof does not rely on the existence of a Green current. Instead, it uses a quantitative estimate on the speed of convergence of preimages of points toward μ. This approach also allows us to treat automorphisms of K\"ahler manifolds.

A brief survey on local holomorphic dynamics

Feng Rong (Shanghai Jiao Tong University)

The local holomorphic dynamics studies the behavior of holomorphic maps under iteration near a fixed point (or set). In this talk, we give a brief survey on this topic, focusing on the generalizations of the well-known Leau-Fatou Flower Theorem to higher dimensions. The theory is wide open in dimensions three and higher.

Combinatorics of Hamiltonian Normal Forms

Dmitry Treschev (Steklov Mathematical Institute, Russian Academy of Sciences)

We discuss algebraic and combinatorial aspects of the Hamiltonian normal form theory. The objective is the description of the normal form near a singular point purely in terms of the original Hamiltonian, avoiding the normalization procedure. In the case of one degree of freedom we compute the normal form as an explicit nonlinear functional, applied to the original Hamiltonian. We present analogous results in arbitrary dimension. The corresponding formulas are more complicated but still explicit.

Vector bundles on Riemann surfaces and isomonodromic deformations

Ilya Vyugin (Higher School of Economics)

We study holomorphic vector bundles with logarithmic connections on a Riemann surface of an arbitrary genus. We present the new bound for slopes of the associated factors of the Harder-Narasimhan filtration and apply it to the study of singularities of isomonodromic deformations of logarithmic connections on the elliptic curve. Solvability of generalized Riemann-Hilbert problem on a curve of positive genus will also be studied.

A dynamical approach to Smale's mean value conjecture

Yuefei Wang (Shenzhen University)

In this talk we present a dynamical study of Smale's mean value conjecture on computational complexity. It is shown that the conjecture holds for several important subclasses of polynomials: almost every polynomial (in the sense of bifurcation measure), postcritically finite maps, the shift locus, and stable algebraic subfamilies, by exploiting results and methodologies in complex dynamical systems.

Divergence of geometric normalizations for elliptic fixed points in the plane

Qiaoling Wei (Capital Normal University)

Classically, for a local analytic diffeomorphism F of (R^2,0) with a non-resonant elliptic fixed point (eigenvalues exp(\pm2\pi i\omega) with \omega real irrational), one can find formal normalizations, i.e. formal conjugacies to a formal diffeomorphism invariant under the group of rotations. Less demanding is the notion of a "geometric normalization" that we introduce: this is a formal conjugacy to a formal diffeomorphism which maps any circle centered at 0 to a circle centered at 0. Geometric normalizations are not unique, but they correspond in a natural way to a unique formal invariant foliation. We then show that, generically, all geometric normalizations are divergent, so there is no analytic invariant foliation. The talk is based on joint works with Alain Chenciner, David Sauzin.

Hyperbolic geodesic flow and spectral theory

Disheng Xu (Great Bay University)

In this talk I will present a recent joint work with Zhenfu Wang and Qi Zhou. We study the spectral theory for long range operators. In particular, we show any finite-range perturbation of a subcritical almost Mathieu operator, with Diophantine frequency, retains purely absolutely continuous spectrum for all phases. My talk will focus on the part that partiallly inspired by the study of hyperbolic geodesic flows.

The Denseness Problem of Uniformly Hyperbolic Systems in analytic Quasiperiodic Cocycles

Jiangong You (Nankai University)

Avila-Jitomirskaya proved that uniformly hyperbolic systems are dense in the one-parameter family of Schrödinger cocycles with the potential $\lambda \cos x$. We will show that uniformly hyperbolic systems are dense in the one-parameter family of Schrödinger cocycles for all Type I potentials, a open class of analytic potentials including $\lambda \cos x$ and their perturbations

（Based on joint works with Lingrui Ge, Svetlana Jitomirskaya and Qi Zhou）